Stochastic Modeling of the Decay Dynamics of Online Social Networks

Mohammed Abufouda and Katharina A. Zweig

Abstract The dynamics of online social networks (OSNs) involves a complicated mixture of growth and decay. In the last decade, many online social networks, like MySpace and Orkut, suffered from decay until they were too small to sustain themselves. Thus, understanding this decay process is crucial for many scenarios that include: (1) Engineering a resilient network, (2) Accelerating the disruption of malicious network structures, and (3) Predicting users' leave dynamics. In this work we are interested in modeling and understanding the decay dynamics in OSNs to handle the aforementioned three scenarios. Here, we present a probabilistic model that captures the dynamics of the social decay due to the inactivity of the members in a social network. The model is proved mathematically to have *submodularity* property. We provide preliminary results and analyse some properties of real networks under decay process and compare it to the model's results. The results show, at the macro level of the networks, that there is a match between the properties of the decaying real networks and the model.

1 Introduction

Today's online social networks represent a main source of communication and information exchange among people all over the world. Many online social networks have proven their usefulness, like Facebook, Twitter, and Linkedin, in connecting people and facilitating an exquisite new medium for sharing news, forming groups of people of the same interests, and eliciting knowledge. The growth of these networks in terms of user activity shows that these online social networks have become a vital part in today's human activities. One well-studied aspect of online social networks dynamics is the *growth* dynamics of a network. The work by Barabási et al. [6] presented a simple model for understanding the growth dynamics of a network, namely the *Preferential Attachment Model* (PAM), which is a rich-get-richer-model. Jin et al. [15] noticed that the model by Barabási et al. [6] and other similar models, like the work by Dorogovtsev et al. [11] for modeling the growth of ran-

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dom networks, are not suitable to understand the growth dynamics of social networks. Thus, they provided a model that considered the specialty of social networks without a power law distribution and with large clustering coefficient [15]. With the availability of the online datasets, Newman [25] studied empirically the growth of social networks using the scientific collaboration networks against the PAM model [6]. Bala et al. [5] provided a non-cooperative game based model for the network formation. Later, Jackson [14] surveyed the models and methods that were used to capture the network formation process and compared them in terms of stability and efficiency. Leskovec et al. [21] first showed on dynamic network data, that networks densify over time and that their diameter is shrinking. They also provided another growth dynamics model that was able to produce networks with these properties. The previous work and the availability of rich datasets pushed the research to an in-depth investigation of the properties of the networks over time. Kumar et al. [20] studied the growth of a large social network in terms of network component analysis, Kossinets et al. [18] studied the tie formation process within the social networks that is affected by internal and external factors, and Capocci et al. [9] studied the statistical properties of the growth characteristics of Wikipedia collaboration social networks. Likewise, Backstrom et al. [4] studied empirically how groups are formed and evolved over time in MySpace social networks , while Mislove et al. [23] provided a study for the growth of Flicker social network. Even though there are many successful social networks, the evolution of a social network also incorporates *decay*. In the last decade, some of the online social networks were closed after a huge loss or inactivity of their members. Online social networks, like Friendsfeed, Friendster, MySpace, Orkut, and many websites of the Stack Exchange platform, are now out of service, despite the fact that some of them, e.g., Orkut and Myspace, showed a tremendous growth [2] just a decade ago. The decay of these networks poses many questions about the reasons behind their fall down. Garcia et al. [12] and Chhabra et al. [10] studied the static properties of Friendster and MySpace, respectively, in order to understand the network-related properties of these networks as an example of a decayed network. Recent studies by Malliaros et al. [22] and Bhawalkar et al. [8] provided theoretical models for understanding the social engagement in online social networks with a potential to predict social inactivity. Torkjazi et al [28] provided an analysis of Myspace online social network and examined the activity and inactivity of its users with some insights about the reasons behind the fall of MySpace. Similarly, Ribeiro [26] studied activity and inactivity of the users by providing a model that uses the number of daily active users as a proxy of the dynamics in the membership based websites. Kairam et al. [16] provided machine learning prediction models to predict community *longevity*: how long a community in an online social network will survive. Another related work done by Asur et al. [3] discussed the persistence and decay of Twitter tweets. While investigating the reasons behind the inactivity of members of an online social networks is not in the scope of this work, some recent studies proposed some answers [27, 17], suggesting that the main reason behind this decay is the inactivity of the members of the online social networks.

Building a sound understanding of the decay dynamics of networks requires not only studying the static properties of these networks, but also requires investigating their dynamics and properties over time, and this is what we are interested in here. As a scenario, we consider the Stack Exchange websites that were closed after some period of time due to the lack of enough activity required to keep the website alive. The closed websites are an example of the social network decay, where we model the members of a website as the nodes of the network and an edge exists between any two nodes if they post, comment, or answer to the same question in the websites.

While we cannot answer why a person starts losing interest in a social network, we can try to analyze and model the effect of this behavior on other people. Such a model might in turn hint at

the causes of social decay or at least explain some part of it.
In this work, we provide a probabilistic model for understanding the social decay phenomenon in online social networks. The model presented here can provide insights regarding the effect of node leave on the neighborhood nodes. Our contribution in this work is split the following: (1) A longitudinal network analysis of the stack exchange sites showing their decay. (2) A probabilistic model for social network decay which is a *step by step* mechanistic model for a node leave and the effect of its leave. (3) Theoretical proof of the submodularity of the model that leads to viable optimization, e.g., determining the minimal set of nodes to leave the network for accelerating/decelerating the decay process. Being submodular renders the maximization problem of the model to be viable.

2 Model and notations

A network $G = (V, E)$ is a tuple of two sets V and E, where V is the set of nodes and E is the set of edges such that an undirected edge e is defined as $e = \{u, v\} \in E$, where $u, v \in V$. As we consider a dynamic system, the notation G^t is a network at time t. We assume that every node $w \in V$ has an initial *Leave Probability* $\pi_w^{t=0}$ which denotes the probability of node w leaving the network at time 1, and generally at $t+1$. If a node w did not leave at $t+1$, i.e., $w \in V(G^{t+1})$, then its current leave probability, π_w^t, will be increased depending on its neighbors who left at $t-1$. The *tie strength* at time $t-1$, representing some possibly dynamic measure of closeness of a relationship, is denoted by $\delta_{v,w}^{t-1}$ and assumed to be $\in (0, 1]$. The details of this process are described in the following sections.

Definition 1. A dynamic network G is called a "Decaying Network" if $|E(G)^{t-1}| \geq |E(G)^t|$, $|V(G)^{t-1}| \geq |V(G)^t|$, and $V(G)^t \subseteq V(G)^{t-1}, \forall t > 0$.

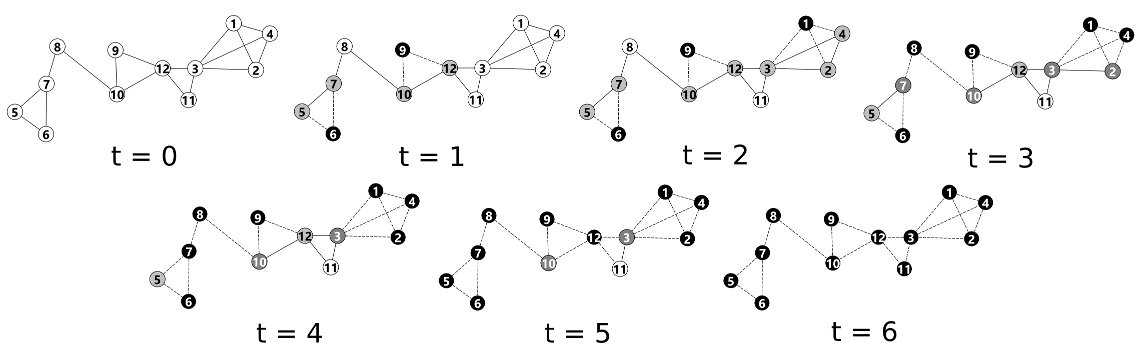


Fig. 1: An illustration of the model. The color of the nodes represents how likely a node will leave in the future, where white nodes are very unlikely to leave and the level of grayness correlates with the probability to leave. Whenever a node leaves the network it is marked as black, all its edges are removed, and all of its neighbors get affected by its leave by increasing their leave probability. The dotted edges are the removed edges.

We assume the model starts with a *Decaying Network*, i.e, no further nodes or edges are added to the network. The main idea of the model is shown in Figure 1.

2.1 Probability Gain

At any point of time t where $t>0$, the node's leave probability changes from π_w^{t-1} to π_w^t, by adding *Probability Gain* $\Delta\pi_w^t$, that never exceeds the value of 1. Thus, a node w will leave at time $t+1$ with probability π_w^{t+1} such that:

$$\pi_w^{t+1} = min\{1,\ \pi_w^{t-1} + \Delta\pi_w^t\} \tag{1}$$

If a node w did not leave the network at time t, then we have two sets: $\overline{\Gamma}_w^{t-1}$ and $\underline{\Gamma}_w^{t-1}$, which are the sets of w's neighbors who left and did not leave the network at $t-1$, respectively.

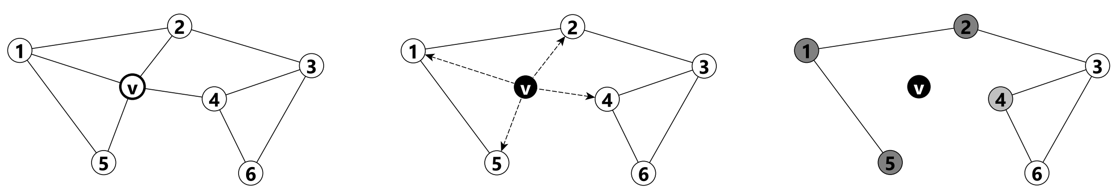
2.1.1 Probability gain due to one node leave:

We first define the probability gain due to the leave of a single neighbor v of the node w at time point $t-1$, and then generalize it to w's neighbors that left the network: $\overline{\Gamma}_w^{t-1}$. Now, the probability gain that a node w will get at $t+1$ due to the leave of its neighbor node v at $t-1$ is defined as:

$$\Delta\pi_w^{t+1}(v) = 1 - (1-\pi_v^{t-1})(1-\delta_{v,w}^{t-1}) \tag{2}$$

where the edge $e=(v,w) \in E(G)_{t-2}$ and $e=(v,w) \notin E(G)_{t-1}$ as $v \in \overline{\Gamma}_w^{t-1}$ and $w \in V(G)_{t-1}$. Thus, the total probability gain produced by the leave of node v to all of its neighbors which did not leave, see Figure 2 for an illustration, is given by:

$$\Delta\pi^t(v) = \sum_{w \in \underline{\Gamma}_v^{t-1}} 1 - (1-\pi_v^{t-1})(1-\delta_{v,w}^{t-1}) \tag{3}$$



$t-2$ $t-1$ t

Fig. 2: This figure shows how a node v affects all of its neighbors when it leaves. At $t-2$, the node v has a leave probability π_v^{t-2} which was gained by v's initial leave probability π_v^0 and possible probability gains caused earlier by leaving neighbors, i.e., $\pi_v^{t-2} = \pi_v^0 + \sum_{t=1}^{t=t-3} \Delta\pi_v^t$. At time $t-1$, the node v leaves the network affecting its neighbors by increasing the leave probability of nodes $1, 2, 4, 5$. Here we assume that the tie strength between v and the nodes $1, 2, 5$ is greater than the tie strength between v and 4. That is why the nodes $1, 2, 5$ gain more leave probability than node 4, which is represented by a darker color of nodes $1, 2, 5$.

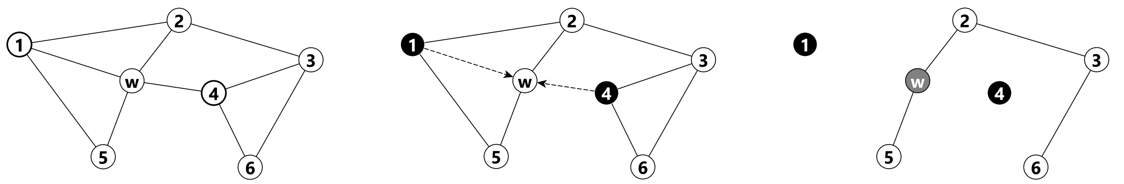


Fig. 3: This figure shows how a node w is affected by the leave of its neighbors. At $t-2$, the nodes $1, 4$ have leave probabilities π_1^{t-2} and π_4^{t-2}, respectively, which were gained by the nodes' initial leave probabilities π_1^0 and π_4^0 and possible earlier probability gains. At time $t-1$, the nodes $1, 4$ leaves the network affecting their neighbors, here we are interested in the node w. The leave of nodes $1, 4$ left node w with an increased leave probability at t. Note that nodes $2, 3, 5, 6$ are affected also by the leave of $1, 4$, but for simplicity and for visualization traceability we concentrated on node w.

2.1.2 Probability gain due to multiple nodes leave:

We now generalize the probability gain induced by the leave of a single node to capture the impact of all neighbors that left, i.e., $\overline{\Gamma}_w^{t-1}$.

$$\begin{aligned}\Delta\pi_w^t &= 1 - [\underbrace{(1-\xi_w^{t-1})}_{\text{Assures leave}}\underbrace{(\prod_{u\in\overline{\Gamma}_w^{t-1}}(1-\pi_u^{t-1}))}_{\text{Leave probabilities effect}}\underbrace{(\prod_{u\in\overline{\Gamma}_w^{t-1}}(1-\delta_{u,w}^{t-1}))}_{\text{Tie strength effect}}] \\ &= 1-[(1-\xi_w^{t-1})(\prod_{u\in\overline{\Gamma}_w^{t-1}}(1-\pi_u^{t-1})(1-\delta_{u,w}^{t-1}))]\end{aligned} \tag{4}$$

where $\xi_w^{t-1} = \frac{|\overline{\Gamma}_w^{t-1}|}{|\Gamma_w^{t-1}|}$ and the quantity $1-\xi_w^{t-1}$ assures that when all of the neighbors of the node w leaves, then the node w will (be forced to) leave too as it will be disconnected. Thus, Equation 1 becomes:

$$\pi_w^t = min\{1,\ \pi_w^{t-1} + 1 - [(1-\xi_w^{t-1})(\prod_{u\in\overline{\Gamma}_w^{t-1}}(1-\pi_u^{t-1})(1-\delta_{u,w}^{t-1}))]\} \tag{5}$$

3 Monotonicity and submodularity

In this section, we show the monotonicity and submodularity properties of the model's equations [1].

Definition 2. Let $f : 2^V \to \mathbb{R}_{\geq 0}$, where $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} \mid x \geq 0\}$, be an arbitrary function that maps the subsets S and T to a non-negative real value, where $S \subseteq T \subset V$. Then, the function f is submodular [19] if it satisfies the following inequality: $f(S \cup \{v\}) - f(S) \geq f(T \cup \{v\}) - f(T)$, where $v \in V \setminus T$.

Lemma 1 (Order preserving of the probability gain sum). *Let $\pi^t = \{\pi_1, \pi_2, \cdots, \pi_n\}$, where $\pi_i \in \pi^t$ and $\pi_i \in (0, 1]$. Then we have: $\sum_{\pi_i\in\pi^t}\pi_i \leq \sum_{\pi_i\in\pi^{t+1}}\pi_i$ where $\pi^t \subseteq \pi^{t+1}$, and the sets π^t and π^{t+1} are defined like above.*

[1] Detailed proofs are provided in an earlier technical paper [1].

Lemma 2 (Order preserving of the probability gain product). *Let* $\pi^t = \{\pi_1, \pi_2, \cdots, \pi_n\}$*, where* $\pi_i \in \pi^t$ *and* $\pi_i \in (0, 1]$*. Then we have:* $\prod_{\pi_i \in \pi^t} \pi_i \geq \prod_{\pi_i \in \pi^{t+1}} \pi_i$ *where* $\pi^t \subseteq \pi^{t+1}$*, and the sets* π^t *and* π^{t+1} *are defined like above.*

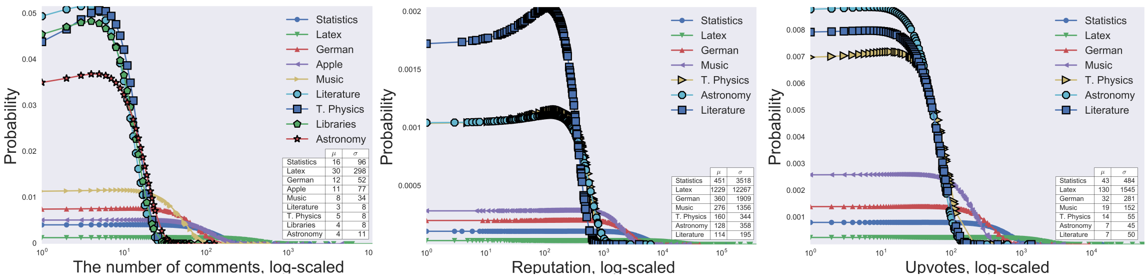
Theorem 1. *The leave probability gain function, Equation 3, is submodular.*
The interpretation of the theorem is that, the more friends a node v had before leaving, the higher its total induced leave probability gain.

Theorem 2. *The leave probability gain function, Equation 4, is monotone, i.e., for a node* w *we have* $\pi_w^t \leq \pi_w^{t+1}$ *if the node* w *did not leave the network at* $t + 1$*.*

Theorem 3. *The leave probability gain function, Equation 4, is submodular.*
The theorem state that the more of your friends leave, the less important the others become. Submodulariy entails an interesting properties: the minimization problem of submodular function can be performed in polynomial time [13], and the maximization problem of the submodular function, which is NP-Hard problem, can be approximated within a factor of $\alpha = (1 - 1/e)$ using a greedy algorithm [24].

4 Results

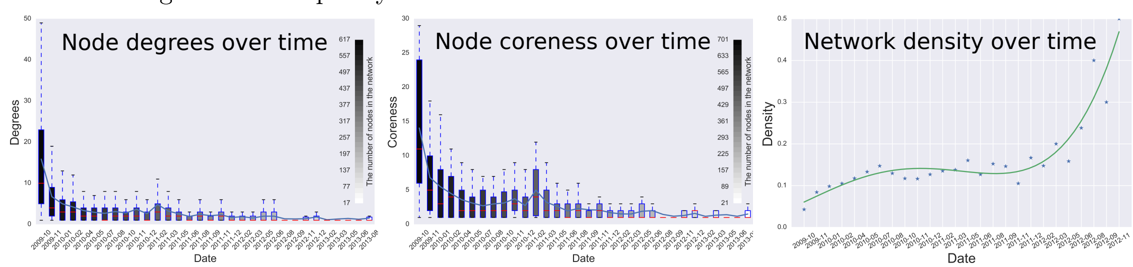
In this section, we provide the analysis of the decaying stack exchange websites and the results of the model. Figure 4a shows the distribution of the number of user comments for alive and decayed websites. The figure shows that the decayed websites clearly have different distribution characteristics with a low mean and low standard deviation. A similar behavior is found in Figure 4b and Figure 4c that represents the distribution of users' total received *Reputation* and *Upvotes*, respectively. These two properties reflect the level of knowledge and experience that the members of a website have. For the decayed websites, it is clear that, on average, the members have much less reputation and upvotes than those in the alive websites. The three figures, Figures 4a, 4b, and 4c show that there is less social activity in the decayed websites, which may be used as an indication for studying the future of the alive websites. However, understanding the decay dynamics of the decayed websites requires a deeper investigation and modeling for the nature of the interaction among the members. Our approach to better understand what happens during the decay process is to make a network representation of the members' interactions, like comments, upvotes, and posts, as networks. Then, we build a network based model for modeling the decay process.



(a) Comments per user. (b) Reputation per user. (c) Upvotes per user.

Fig. 4: (Color Online) The characteristics of the interaction decay in the decayed and alive websites of the Stack Exchange websites. The figures show the probability distributions of different types of interactions in these websites. Markers with bold boarders are decayed websites, μ is the mean, and σ is the standard deviation. From the figures it is clear that the decayed networks have different distribution properties from the other alive networks.

Algorithm 1 depicts the steps we followed in our experiments. Line 4 initializes the initial leave proability π_v^0, which is a design decision and we selected values from 0.0005 to 0.045 with an 0.0005 increase step. For each of these values, the model runs and simulates Equation 4. The update step in line 13 simulates Equation 5. The result of the algorithm is a set of graphs that are used for the analysis. The output of this algorithm results in a large number of graphs. For example in the case of the Startup Business website we have analyzed more than $200k$ graphs with 250 runs for each probability to get more confidence of the results. The tie strength was a normalized edge weight where the weight is the frequency of the interaction between two nodes.



(a) (b) (c)

Fig. 5: (Color Online) Macro properties of the real networks under decay for the Startup business site. Figures 5.a, 5.b, 5.c show the degrees of the nodes, the node coreness, and the network density over time.

In Figure 5 we show the macro properties of the real networks of the Startup Buisness website over time. The network evolution shows a clear decay that is represented as a decrease in the number of the nodes. This decrease was associated with a decrease in the average degrees of the nodes over time and also with a decrease of the node's coreness [7]. Another macro measure we used is the network density. Figure 5c shows an increase in the density over time. This increase is due to early leave of the nodes with less degrees, i.e., the nodes that are part of dense subgraphs seem to leave the network late. Now, we will show the results of the model simulation. Figure 6a shows the number of components in the network over simulation for different values of π_v^0. The number of components start to increase to a maximum value before it start to decrease. The reason is that at the beginning the model starts with a one-connected component graph and after each step some nodes are removed due to the leave probability. The leave of some nodes results in a disconnected graph with more components. The number of these disconnected components increases until these disconnected components are composed of only triples or simple edges. As a result, a node that leaves from these triples or from these edges will not increase the number of the components anymore. Figure 6b and Figure 6c show a similar behaviour for the average degree and the average coreness over time, respectively. The more nodes are being removed from the network, the less edges remain and thus the average degree and the average coreness decrease uniformly over time. This behavior of the model is similar to the real data presented in Figure 5. The last global measure that we use is the network density as shown in Figure 6d. The density of the simulated networks increases over time for the same reason stated for the real networks in Figure 5. These results show that the model provides a real-like behaviour of the networks under decay.

Algorithm 1 Model simulation

Input: Graph G_0
Output: Graphs= $\{G_0, G_1, \cdots, G_{n-1}\}$ where G_n is an empty graph
for all $v \in V(G_0)$ **do**
    initialize π_v^0
$t = 0, G_t = G_0$, Graphs.add(G_t)
while G_t is not empty **do**
    LeftNodes$_t = \emptyset$
    $t = t + 1$
    for $v \in V(G_t)$ **do**
        if Leave(v,π_v^t) is $True$ **then**
            LeftNodes$_t$.Add(v)
    for all $u \notin$ LeftNodes & $\overline{\Gamma}_u^{t-1} \neq \emptyset$ **do**
        update(π_u^t,$\overline{\Gamma}_u^{t-1}$)
    remove LeftNodes$_t$ from G_t
    Graphs.add(G_t)

5 Discussion

There are different applications where the model can be utilized. *1. Social network resilience:* the resilience against huge disruptions in social networks is not well-studied. We think that the model provides a first step towards engineering a resilient social network via understanding the decay dynamics of a network. *2. Leave cascade detection*: the leave of one member is not as harmful as a cascade of leaves for the networks that seek growth. The model captures the dynamics of leave cascades by observing the leave probabilities of the nodes and their increase. *3. Maximizing the leave effect:* for a network where a dissolving process is required, like criminal social networks, the model is able to provide a viable disruption maximization (thanks to the submodularity property of the model) to the network with insights about the influential members and the effect of the leave.

6 Conclusion

In this work, we presented an empirical analysis of the social decay dynamics of the closed Stack Exchange websites. The closed websites showed an inactivity, which might have caused their decay. We model these interactions between the members of these websites as a network that enabled us to

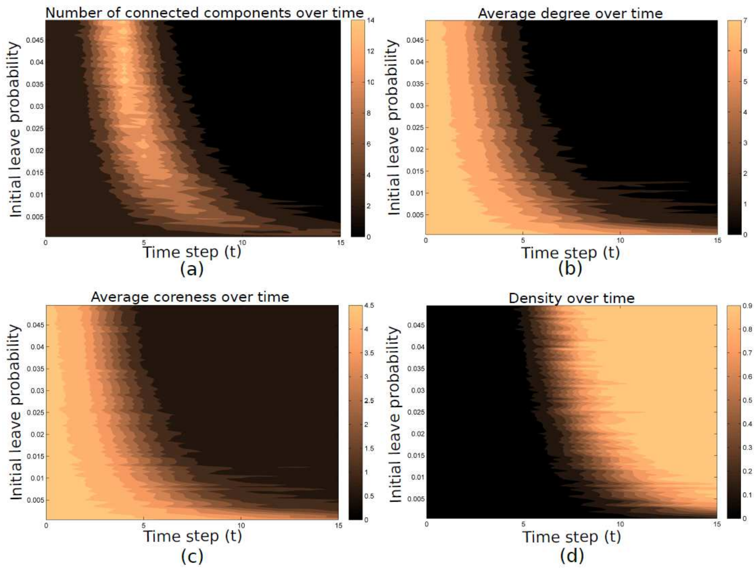


Fig. 6: (Color Online) The results of multiple global measures of the model. Figure 6a, Figure 6b, Figure 6c, and Figure 6d show the number of components, the average degree, the average coreness, and the density of the network over time for different values of initial leave probability π_v^0, respectively. The model started with G_0 as the input network and simulates the decay over it.

build a model to understand the decay dynamics. Then, we have presented a model for capturing the decay dynamics in social networks. The model is a probabilistic model that assumes that the leave of social network members affects the leave of their neighbors. In this work we have also presented some mathematical properties and proved them. We proved that the model's main equations are submodular, which entails doing optimization of the model in a feasible way. Also, we presented the macro network properties of real networks under decay and compared these results with the results of the model simulation. The results of the model and the real networks under decay showed a similar behavior that supports the potential of the model for different usages. In the future, we will design the optimization algorithms and study the applicability of the model and also provide more empirical validation of its properties.

References

1. M. Abufouda and K. A. Zweig. A theoretical model for understanding the dynamics of online social networks decay. *arXiv preprint arXiv:1610.01538*, 2016.
2. Y.-Y. Ahn, S. Han, H. Kwak, S. Moon, and H. Jeong. Analysis of topological characteristics of huge online social networking services. In *Proceedings of the 16th international conference on WWW*, pages 835–844. ACM, 2007.
3. S. Asur, B. A. Huberman, G. Szabo, and C. Wang. Trends in social media: Persistence and decay. *Available at SSRN 1755748*, 2011.
4. L. Backstrom, D. Huttenlocher, J. Kleinberg, and X. Lan. Group formation in large social networks: membership, growth, and evolution. In *Proceedings of the 12th ACM SIGKDD*, pages 44–54. ACM, 2006.
5. V. Bala and S. Goyal. A noncooperative model of network formation. *Econometrica*, 68(5):1181–1229, 2000.

6. A.-L. Barabási and R. Albert. Emergence of scaling in random networks. *American Association for the Advancement of Science*, 286(5439):509–512, 1999.
7. V. Batagelj and M. Zaversnik. An o(m) algorithm for cores decomposition of networks. *arXiv preprint cs/0310049*, 2003.
8. K. Bhawalkar, J. Kleinberg, K. Lewi, T. Roughgarden, and A. Sharma. Preventing unraveling in social networks: the anchored k-core problem. *SIAM Journal on Discrete Mathematics*, 29(3):1452–1475, 2015.
9. A. Capocci and et al. Preferential attachment in the growth of social networks: The internet encyclopedia wikipedia. *Physical Review E*, 74(3):036116, 2006.
10. S. S. Chhabra, A. Brundavanam, and S. Shannigrahi. An alternative explanation for the rise and fall of myspace. *CoRR*, abs/1403.5617, 2014.
11. S. N. Dorogovtsev and J. F. F. Mendes. Scaling behaviour of developing and decaying networks. *EPL (Europhysics Letters)*, 52(1):33, 2000.
12. D. Garcia, P. Mavrodiev, and F. Schweitzer. Social resilience in online communities: The autopsy of friendster. In *Proceedings of the first ACM conference on Online social networks*, pages 39–50. ACM, 2013.
13. S. Iwata, L. Fleischer, and S. Fujishige. A combinatorial strongly polynomial algorithm for minimizing submodular functions. *Journal of the ACM (JACM)*, 48(4):761–777, 2001.
14. M. O. Jackson. A survey of network formation models: stability and efficiency. *Group Formation in Economics: Networks, Clubs, and Coalitions*, pages 11–49, 2003.
15. E. M. Jin, M. Girvan, and M. E. Newman. Structure of growing social networks. *Physical review E*, 64(4), 2001.
16. S. R. Kairam, D. J. Wang, and J. Leskovec. The life and death of online groups: Predicting group growth and longevity. In *Proceedings of the fifth international conference on Web search and data mining*, pages 673–682. ACM, 2012.
17. A. A. Kordestani, M. Limayem, E. Salehi-Sangari, H. Blomgren, and A. Afsharipour. Why a few social networking sites succeed while many fail. In *The Sustainable Global Marketplace*, pages 283–285. Springer, 2015.
18. G. Kossinets and D. J. Watts. Empirical analysis of an evolving social network. *science*, 311(5757):88–90, 2006.
19. A. Krause and D. Golovin. Submodular function maximization. *Tractability: Practical Approaches to Hard Problems*, 2012.
20. R. Kumar, J. Novak, and A. Tomkins. Structure and evolution of online social networks. In *Proceedings of the 12th ACM SIGKDD Inte. Conference on Knowledge Discovery and Data Mining*, pages 611–617, 2006.
21. J. Leskovec, J. Kleinberg, and C. Faloutsos. Graphs over time: densification laws, shrinking diameters and possible explanations. In *Proceedings of the eleventh ACM SIGKDD*, pages 177–187. ACM, 2005.
22. F. D. Malliaros and M. Vazirgiannis. To stay or not to stay: modeling engagement dynamics in social graphs. In *Proceedings of the 22nd ACM inter. conference on Information & Knowledge Management*, pages 469–478, 2013.
23. A. Mislove, H. S. Koppula, K. P. Gummadi, P. Druschel, and B. Bhattacharjee. Growth of the flickr social network. In *Proceedings of the first workshop on Online social networks*, pages 25–30. ACM, 2008.
24. G. L. Nemhauser and L. A. Wolsey. Best algorithms for approximating the maximum of a submodular set function. *Mathematics of operations research*, 3(3):177–188, 1978.
25. M. E. Newman. Clustering and preferential attachment in growing networks. *Physical review E*, 64(2):025102, 2001.
26. B. Ribeiro. Modeling and predicting the growth and death of membership-based websites. In *Proceedings of the 23rd international conference on World Wide Web*, pages 653–664. ACM, 2014.
27. S. Stieger, C. Burger, M. Bohn, and M. Voracek. Who commits virtual identity suicide? differences in privacy concerns, internet addiction, and personality between facebook users and quitters. *Cyberpsychology, Behavior, and Social Networking*, 16(9):629–634, 2013.
28. M. Torkjazi, R. Rejaie, and W. Willinger. Hot today, gone tomorrow: on the migration of myspace users. In *Proceedings of the 2nd ACM workshop on Online social networks*, pages 43–48. ACM, 2009.